Paper Reference 9MA0/02
Pearson Edexcel
Level 3 GCE

Mathematics
Advanced
Paper 2: Pure Mathematics 2

Wednesday 12 June 2019 – Morning

Time: 2 hours plus your additional time allowance.

MATERIALS REQUIRED FOR EXAMINATION
Mathematical Formulae and Statistical Tables
Calculator

ITEMS INCLUDED WITH QUESTION PAPER
Diagram Book
Answer Book

Y58354A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Book and on the Diagram Book, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Book or on the separate diagrams − there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **14** questions in this Question Paper.

The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the Answer Book.

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x

(Total for Question 1 is 3 marks)

---

2. Refer to the table for Question 2 in the Diagram Book.

The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table with the time in seconds and the speed in $m\,s^{-1}$

Using all of this information,

(a) estimate the length of runway used by the jet to take off.
(3 marks)

(continued on the next page)

Turn over

2. continued.

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.
(1 mark)

(Total for Question 2 is 4 marks)

3. Refer to the diagram for Question 3 in the Diagram Book.

It shows a sector AOB of a circle with centre O, radius 5 cm and angle AOB = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500 \text{ cm}^2$$

(continued on the next page)

3. continued.

(a) Explain the error made by this student.

(1 mark)

(b) Write out a correct solution.

(2 marks)

(Total for Question 3 is 3 marks)

4. **Refer to the diagram for Question 4 in the Diagram Book.**

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leq t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in the diagram.

(continued on the next page)

4. continued.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S

(Total for Question 4 is 6 marks)

5. **Refer to the diagram for Question 5 in the Diagram Book.**

It shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on the diagram, has height y and width δx

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\,\delta x$$

(Total for Question 5 is 3 marks)

---

6. Refer to the diagram for Question 6 in the Diagram Book.

It shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x - 2)^2 + 1 & x \leq 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$

(2 marks)

(b) Find all values of x for which $g(x) > 28$

(4 marks)

(continued on the next page)

6. continued.

The function h is defined by

$$h(x) = (x - 2)^2 + 1 \qquad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1 mark)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3 marks)

(Total for Question 6 is 10 marks)

---

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model. (1 mark)

(continued on the next page)

7. continued.

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that

$$y = 0{\cdot}84x + 428$$

(3 marks)

(continued on the next page)

7. continued.

(c) With reference to the model, interpret the significance of the value $0 \cdot 84$ in the equation.
(1 mark)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.
(2 marks)

(Total for Question 7 is 7 marks)

---

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3 marks)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3 marks)

(Total for Question 8 is 6 marks)

---

9. Refer to the diagrams for Question 9 in the Diagram Book.

A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

(continued on the next page)

9. continued.

The results are shown together with a data point from each graph in the Diagram Book.

(a) Explain how Diagram 2 would lead the engineer to believe that the braking distance should be modelled by the formula

$d = kV^n$ where k and n are constants

with $k \approx 0{\cdot}017$

(3 marks)

(continued on the next page)

9. continued.

Using the information given in Diagram 1, with $k = 0 \cdot 017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3 marks)

(continued on the next page)

9. continued.

Sean is driving this car at $60\ \mathrm{km\ h}^{-1}$ in wet conditions when he notices a large puddle in the road 100 metres ahead.

It takes him $0 \cdot 8$ seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.
(3 marks)

(Total for Question 9 is 9 marks)

10. Refer to the diagram for Question 10 in the Diagram Book.
It shows a sketch of triangle OAB

The point C is such that

$\overrightarrow{OC} = 2\overrightarrow{OA}$

The point M is the midpoint of AB

The straight line through C and M cuts OB at the point N

Given $\overrightarrow{OA} = \underline{\mathbf{a}}$ **and** $\overrightarrow{OB} = \underline{\mathbf{b}}$

(a) Find $\overrightarrow{CM}$ **in terms of** $\underline{\mathbf{a}}$ **and** $\underline{\mathbf{b}}$

(2 marks)

(continued on the next page)

Turn over

10. continued.

(b) Show that

$$\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\underline{a} + \frac{1}{2}\lambda\underline{b},$$

where λ is a scalar constant.

(2 marks)

(c) Hence prove that $ON : NB = 2 : 1$

(2 marks)

(Total for Question 10 is 6 marks)

11. Refer to the diagram for Question 11 in the Diagram Book.

It shows a sketch of the curve C with equation

$y = x^x, \quad x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5 marks)

(continued on the next page)

Turn over

11. continued.

The point $P(\alpha, 2)$ lies on C

(b) Show that

$$1 \cdot 5 < \alpha < 1 \cdot 6$$

(2 marks)

(continued on the next page)

11. continued.

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{\,1-x_n}$$

Using this formula with $x_1 = 1\cdot5$

(c) find x_4 to 3 decimal places,
(2 marks)

(d) describe the long–term behaviour of x_n
(2 marks)

(Total for Question 11 is 11 marks)

12. (a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$$

$\theta \neq (90n)°, n \in \mathbb{Z}$

(4 marks)

(b) Hence solve, for $90° < \theta < 180°$,

the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to

one decimal place.

(3 marks)

(Total for Question 12 is 7 marks)

---

13. Refer to the diagram for Question 13 in the Diagram Book.

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in the diagram.

The walls of the tank are assumed to have negligible thickness.

(continued on the next page)

13. continued.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\ m^3$

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4 marks)

(continued on the next page)

13. continued.

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.
(4 marks)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.
(2 marks)

(Total for Question 13 is 10 marks)

---

14. (a) Use the substitution

$u = 4 - \sqrt{h}$ to show that

$$\int \frac{dh}{4 - \sqrt{h}} = -8\ln\left|4 - \sqrt{h}\right| - 2\sqrt{h} + k$$

where k is a constant

(6 marks)

(continued on the next page)

14. continued.

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{dh}{dt} = \frac{t^{0\cdot25}(4-\sqrt{h})}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2 marks)

(continued on the next page)

14. continued.

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.
(7 marks)

(Total for Question 14 is 15 marks)

TOTAL FOR PAPER IS 100 MARKS
END OF PAPER
